EFFECT OF STRAIN RATE ON THE COMPRESSIBILITY AND SHEAR OF SANDS AND CLAYS SUBJECTED TO SHORT-DURATION DYNAMIC LOADS

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In this paper we present the results of experimental investigations of the compressibility and shear of sands and clays subjected to short-duration dynamic loads created by the impact on the sample of a falling weight [1] and the explosion of a TNT charge in a body of undisturbed soil [2-5]. An analysis of the results indicates the importance of the influence of time effects of the viscosity type (influence of strain rate) on the volume compressibility of the investigated soils. At the same time, the possibility of disregarding, in first approximation, the influence of these effects on the shear is noted.

1. Investigation of Soil Compressibility under Laboratory Conditions. In [1] a description was given of a method of conducting laboratory studies of the compressibility of soils under short-duration dynamic loads created by the impact of a falling weight. The laboratory apparatus consisted of a cylinder containing a soil sample (diameter D = 150 mm, height h = 30 mm) in a special ring and a piston for transmitting the impact load to the sample. Various deformation regimes were obtained by varying the tripping height and using different kinds of cushioning. The principal normal stresses – vertical $\sigma_x(t)$ and horizontal $\sigma_y(t)$ – were registered with strain gauges in the center of the piston, in the center and at the edge of the cylinder bottom, and in the sides of the ring. The displacement u(t) of the piston was measured with a high-frequency cantilever-type deflectometer. Since the ratio D/h = 5 is fairly large, we assume a homogeneous strain distribution over the height of the sample and determine the strain from the formula

$$\varepsilon(t) = u(t) / h \qquad (1.1)$$

where h is the original height of the sample.

The instrument readings were amplified and recorded on N-102 and N-105 loop oscillographs.

The investigation of the compressibility of soil samples presupposes a quasi-static loading regime, when wave processes in the sample can be neglected. To estimate the possible errors we calculated the multiple reflection of plastic shock waves in the soil sample in relation to the experimental conditions. Here it was assumed that the soil is characterized by a unique stress-strain relation that is linear during

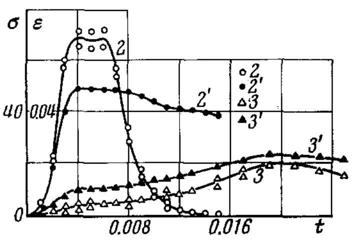

Fig. 1

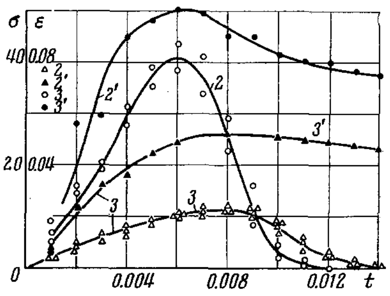

Fig. 2

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 3, pp. 155-160, May-June, 1969. Original article submitted June 29, 1967.

loading. It was also assumed that the particle density does not change during unloading. As a result of these calculations it was found that the error of the quasi-static as compared with the wave approach does not exceed ±15% after a time $t_0 = (5\text{-}6)h/a_0$, where a_0 is the propagation velocity of the shock front. In all the experiments the duration of the process considerably exceeded t_0.

As a rule, the values of the stresses $\sigma_x(t_0)$ did not exceed 5-10% (in individual cases up to 20%) of the maximum values $\sigma_x{}^m$.

By eliminating time, we constructed uniaxial compression diagrams $\sigma = \sigma(\varepsilon)$ from the $\sigma(t) \equiv \sigma_x(t)$ and $\varepsilon(t)$ records. The simultaneous measurement of the two principal measures $\sigma_x(t)$ and $\sigma_y(t)$ under conditions of plane deformation enabled us to construct the yield condition. Each of the $\sigma(\varepsilon)$ diagrams constructed by the method described above corresponded, generally speaking, to a certain arbitrary deformation regime. To evaluate these regimes we used the mean strain rate

$$\dot{\varepsilon}_i = \frac{\varepsilon_{*i}}{t_{*i}} \tag{1.2}$$

Here, ε_{*i} and t_{*i} are the maximum strain and the time required to reach it, respectively; i is the number of the curve.

We tested under laboratory conditions sand with a skeletal density $\gamma_0 = 1.50\text{-}1.55$ g/cm³ and mass moisture content w = 8-10%, undisturbed loess-type loam with $\gamma_0 = 1.35\text{-}1.45$ g/cm³ and moisture content w varying from 3% to 12-14%, and a loam with $\gamma_0 = 1.60\text{-}1.65$ g/cm³ and w = 10-15%. The granulometric composition of these soils is given in [3-5].

The results of analyzing the stress $\sigma(t)$ (points 2 and 3) and strain $\varepsilon(t)$ (points 2' and 3') oscillograms for samples of sand and loam are presented in Figs. 1 and 2, respectively. Here and in what follows the stresses σ are in kg/cm², time t in sec, strain rate $\dot{\varepsilon}_i$ in 1/sec, and rate of change of stress $\dot{\sigma}$ in kg/cm²·sec. The $\sigma(\varepsilon)$ diagrams for sand and loam, constructed by the method described, are presented in Figs. 3 and 4, respectively. Points 2 and 3 (Figs. 3, and 4) correspond to points 2 and 2' and 3 and 3' (Figs. 1 and 2). The mean strain rates for curves 2 and 3 are equal to $\dot{\varepsilon}_2 = 12.2$, $\dot{\varepsilon}_3 = 1.2$ for sand (Fig. 3); and $\dot{\varepsilon}_2 = 15.0$, $\dot{\varepsilon}_3 = 6.5$ for loam (Fig. 4).

Curves 1 (Figs. 3, 4) were constructed for the same soils starting from the relations at the shock front in accordance with the experimental data obtained under field conditions from the underground explosion of TNT charges [2, 4, 5]. Thus, these curves correspond to an infinite strain rate $\dot{\varepsilon}_1 = \infty$ and represent the limit for the given type of soil. Curve 4 in Fig. 3 was constructed on the basis of the results of static tests at $\dot{\varepsilon}_4 = 1 \cdot 10^{-7}$. Similar data for loess-type loams were given previously in [1].

From the data of Figs. 3 and 4 and [1] it is clear that the differences in the strains for stresses $\sigma = 20\text{-}40$ and variations of $\dot{\varepsilon}_i$ on the interval from $\dot{\varepsilon}_1 = \infty$ to $\dot{\varepsilon}_4 = 1 \cdot 10^{-7}$ are 50-100% for the sand and reach 300% or more for the loams. For the sand with $\gamma_0 = 1.60$ g/cm³ and w = 15-16% it has been found [6] that the difference between the static strains and the strains at a mean strain rate $\dot{\varepsilon} = 35$ and $\sigma = 20$ is about 100%. For dry sand with $\gamma_0 = 1.66\text{-}1.75$ g/cm³ it has been found [7, 8] that the maximum differences in the strains under static and instantaneous dynamic loading are 30-50%. It follows from [8] that a difference in the strains of up to 30% persists up to stresses on the order of 50,000 kg/cm².

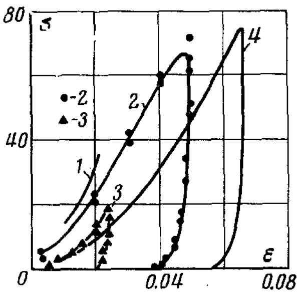

Fig. 3

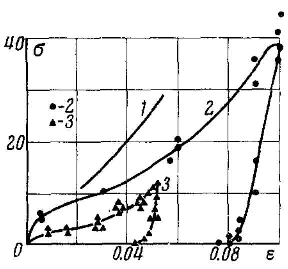

Fig. 4

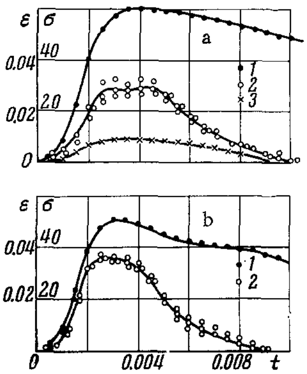

Fig. 5

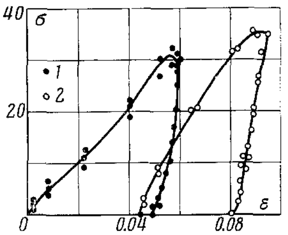

Fig. 6

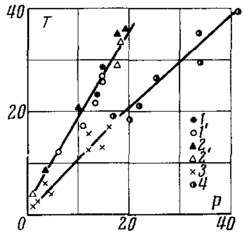

Fig. 7

Thus, an analysis of the experimental results indicates the general importance of the effect of the strain rate on the compressibility of sands and loams under short-duration dynamic loading. As noted in [1], this effect is such that at large strain rates $\dot{\varepsilon} \to \infty$ it disappears, so that there is a limiting form of the relations between the stresses and the deformation characteristics – a limiting dynamic diagram that no longer contains the strain rate. There is also another limiting form of the relations between the stresses and the deformation characteristics – the static diagram corresponding to zero strain rate.

Let us now consider certain experimental results characterizing the effect of strain rate on compressibility during unloading. In the experiments on the compressibility of sands and loams, despite the rather broad range of variation of the mean rate of fall of stress $|\dot{\sigma}_{*i}| = 1 \cdot 10^3 - 1 \cdot 10^5$, it was not possible to achieve values of the mean rate of fall of strain $|\dot{\varepsilon}_{*i}|$ greater than $|\dot{\varepsilon}_{*i}| = 3$-$6$. In particular, for curve 2 (Fig. 1) the quantity $|\dot{\sigma}_2|$ is equal to $|\dot{\sigma}_{*2}| = 1.7 \cdot 10^4$ and coincides with the mean rate of change of stress during loading $\dot{\sigma}_2$, while the absolute value of $\dot{\varepsilon}_{*2}$ is equal to $|\dot{\varepsilon}_{*2}| = 1.4$, which is almost an order less than $\dot{\varepsilon}_{*2}$. For curve 3 (Fig. 1) the mean strain rate during unloading is equal to $|\dot{\varepsilon}_{*3}| = 0.5$. For curves 2' and 3' (Fig. 2) we have $|\dot{\varepsilon}_{*2}| = 4.1$, $|\dot{\varepsilon}_{*3}| = 1.0$. The mean rate $|\dot{\varepsilon}_{*i}|$ in the experiments on loess-type soils did not exceed $|\dot{\varepsilon}_{*i}| = 2$-$3$.

It should be noted that when concentrated charges weighing C = 0.2-200 kg are exploded in soils at maximum stresses $\sigma^m \leq 200$-250 the values of $|\dot{\sigma}_*|$ do not exceed $|\dot{\sigma}_*| = 1 \cdot 10^5$ [2-5]. As the weight of the charge and hence the duration of the blast waves increases, the $|\dot{\sigma}_*|$ decrease. Thus, it may be expected that in connection with the propagation of blast waves in sands and loams at C ≥ 0.2-200 kg the values of $|\dot{\sigma}_*|$ and $|\dot{\varepsilon}_*|$ will likewise not be greater than the above-mentioned limits.

Now, analyzing the $\sigma(\varepsilon)$ diagrams (Figs. 3 and 4) and the analogous results for loess-type loams [1], we may conclude that on the range of variation $|\dot{\sigma}_*| \leq 1 \cdot 10^5$ and $|\dot{\varepsilon}_{*i}| = 1 \cdot 10^{-7}$ (the latter value relates to static tests) the effect of strain rate on compressibility at $\partial\varepsilon/\partial t < 0$ is unimportant.

We also note that in the experiments we observed cases of a further increase of strains in the presence of unloading with respect to the stresses $\partial\sigma/\partial t < 0$. By way of illustration, Figs. 5 and 6 present the experimental results on the compressibility of a loess-type loam with $\gamma_0 = 1.50$ g/cm^3 and w = 10.6% for two successive loadings of the sample.

The experimental points characterizing the change of stress $\sigma(t)$ and strain $\varepsilon(t)$ with time are plotted in Fig. 5a,b for the first (Fig. 5a) and second (Fig. 5b) loadings. Points 1 relate to the strain $\varepsilon(t)$, points 2 to the vertical stresses $\sigma_x(t) \equiv \sigma(t)$, and points 3 to the horizontal stresses $\sigma_y(t)$. The compression $\sigma(\varepsilon)$ diagrams corresponding to Fig. 5 are shown in Fig. 6 for the first (curve 1) and second (curve 2) loadings. It is clear from Fig. 5a,b that in both cases the strains continue to increase for a certain time after satisfaction of the condition $\partial\sigma/\partial t < 0$.

Thus, the usual stress unloading condition $\partial\sigma/\partial t < 0$ will not serve as a criterion of the onset of unloading when the strain rate has an important influence on compressibility and should be replaced by the more general condition $\partial\varepsilon/\partial t < 0$.

Finally, we note two additional experimental facts attesting to the influence of time effects on soil compressibility. In the experiments it was observed that for both sands and loams the repeat loading curve on the $\sigma(\varepsilon)$ diagrams did not coincide with the unloading curve of the first experiments. This is clear, in particular, from Fig. 6, where for the same stress $\sigma = 31$ the repeat loading strain increased by more than 30% as compared with the first loading. In a number of cases, after the stresses had fallen to zero, the strains continued to decrease for a certain time, i.e., there was an aftereffect. The values of the maximum aftereffect strains in loams and loess-type soils did not exceed 10-15% of the residual strains at the instant of disappearance of the load (Figs. 4 and 6).

2. Results of an Experimental Investigation of the Shearing of Soils under Short-Duration Loads. The nature of the shear deformation of a plastic medium is analytically described by the relations between the strain rate deviator (or strain deviator in deformation theories) and the stress deviator. In this case irreversible plastic deformation is possible only when a certain yield condition is satisfied [9-11]. In [11] equations are proposed, especially for soils, in which the strain rate and stress deviators are related in accordance with a flow theory of the Prandtl-Reuss type, while the yield condition has the form of the Mises-Schleicher condition:

$$I_2 = {}^1/_6 F^2(p), \quad p = -{}^1/_3 \sigma_{kk}$$
$$I_2 = {}^1/_2 S_{ij} S_{ij}, \; S_{ij} = \sigma_{ij} + p\delta_{ij} \; (i, j = 1, 2, 3) \tag{2.1}$$

Here, σ_{ij} are the stress tensor components, p is the mean hydrostatic pressure, and F(p) is a certain nondecreasing function of p determined experimentally.

The relation between the deviators in [11] possesses the properties of dry friction. Thus, the soil shear relations in question are homogeneous in time and hence do not take into account the influence of viscosity effects on the shear.

Experimental investigations [2-5] have confirmed the applicability of yield condition (2.1) to sands and clays; the function f(p) was obtained in the linear form

$$F(p) = kp + b \tag{2.2}$$

where k and b are constants characterizing the internal friction and cohesion in the soil:

	k	b, kg/cm^2	ξ_*	ξ_0
(1)	1.23—1.25	0	0.45	0.42
(2)	1.80—1.90	0.40—0.50	0.23—0.29	0.36
(3)	1.70—1.80	1.50—1.70	0.31—0.38	—
(4)	1.00	1.00	0.52	0.50—0.60
(5)	0.95	2.00	0.54	0.55—0.70

Here, rows 1-5 relate:

1) to loose sandy soil with γ_0 = 1.30-1.40 g/cm^3, w = 10-12%;
2) to undisturbed sandy soil with γ_0 = 1.50-1.52 g/cm^3, w = 8-12%;
3) to a loess-type loam [1, 3] with γ_0 = 1.35-1.45 g/cm^3, w = 12-14%;
4) to a loam with γ_0 = 1.60-1.65 g/cm^3, w = 10-15%;
5) to a dense clay with γ_0 = 1.70-1.75 g/cm^3 and w = 20-22%.

The granulometric composition of these soils is given in [3-5]. We note that rows 2 and 4 of the table correspond to the soils considered in section 1.

Relation (2.1) proved to be independent both of the value of the stresses in the range from σ = 1-2 to σ = 250 and of the symmetry conditions [2-4].

Further investigations have shown that the strain rate likewise does not have much effect on yield condition (2.1) during either loading or unloading.

The results of constructing the yield function F(p) on the basis of the data of Fig. 5 for two successive loadings are presented in Fig. 7 (curve 1). Here, along the ordinate axis we have plotted values of $T = \sqrt{6I_2} = \sqrt{2}(\sigma_x - \sigma_y)$, and along the axis of abscissas the mean hydrostatic stress $p = -{}^1/_3(\sigma_x + 2\sigma_y)$. The symbols 1 and 1' relate to loading and unloading (with respect to stresses) during the first loading, the symbols 2 and 2' to loading and unloading during the second loading, respectively. It is clear from Fig. 7 that all the experimental points are closely approximated by a single linear function F(p) at k = 1.80, b = 2.0 kg/cm^2.

Reference [1] gave the results of an investigation of the yield condition for samples of loess-type loams at various mean strain rates, when important differences in the $\sigma(\varepsilon)$ compression diagrams were observed. In those experiments the yield condition again proved to be independent of the strain rate during both loading and unloading.

A similar conclusion was also drawn from an analysis of the results of an investigation of the yield condition in undisturbed sands and clays through which blast waves were propagated [2-5]. In these experiments the strain rate varied within wide limits from $\dot{\varepsilon} = \infty$ at the shock front to $\dot{\varepsilon} \to 0$ during unloading. However, all the results were quite satisfactorily described by a single (for each soil) linear function (2.2). For sandy soils these data are presented in [2, 4], and for loams and clays in Fig. 7. The loams are represented by points 3 and the clays by points 4. The values of the coefficients k and b for these soils are given in rows 4 and 5 above.

To evaluate the role of viscosity in shear, in the general case it is not sufficient to estimate only the effect of the compressive strain rate. It is also necessary to estimate the effect of the shear strain rate. It can be shown, however, that in the given experiments the maximum shear strain rate is of the same order as the compressive strain rate. In particular, for tests under conditions of plane deformation the maximum shear strain rates $\dot{\gamma}_{x'y'}$ occur on elements inclined at an angle $\alpha = {}^1/_4\pi$ to the surface of the sample and are equal to

$$\dot{\gamma}_{x'y'} = -2\dot{\varepsilon}_x \equiv -2\dot{\varepsilon}, \quad \dot{\gamma}_{x'y'} = \frac{\partial v_{x'}}{\partial y'} + \frac{\partial v_{y'}}{\partial x'} \tag{2.3}$$

where $v_{x'}$, $v_{y'}$ are the particle velocities in the new coordinate system x', y'.

Thus, it may be concluded that the shear strain rate likewise does not have an important influence on the yield condition under dynamic loads within the limits of the deformation regimes investigated in the experiments.

Let us compare the data on the yield condition obtained for short-duration loads with the existing lateral pressure coefficients of soil mechanics.

From (2.1), recalling that $p = -{}^1/_3(\sigma_x + 2\sigma_y)$, for plane deformation we obtain

$$\xi(p) = \frac{\sigma_y}{\sigma_x} = \frac{(3\sqrt{2} - k)\,p/b + 1}{(3\sqrt{2} + 2k)\,p/b - 2} \tag{2.4}$$

The asymptotic value of $\xi(p)$ as $p/b \to \infty$ is equal to

$$\xi_* = \frac{3\sqrt{2} - k}{3\sqrt{2} + 2k} \tag{2.5}$$

and is usually called the lateral pressure coefficient. We note that the variability of the lateral pressure coefficient in sandy soils subjected to static loads was first observed by G. I. Pokrovskii.

The table presents values of ξ_* based on the results of a blast-wave investigation of the yield condition together with values of coefficient ξ_0 based on the data of static tests [12-14].

A comparison of ξ_* and ξ_0 shows that for sands and loams they are similar, while for clay ξ_* is somewhat less than ξ_0. Thus, viscosity does not have an important influence on the yield condition even on transition from instantaneous loading to very slow, static loading conditions.

The unimportance of the effect of the strain rate on the yield condition in the general case does not imply the unimportance of the effect on the shear as a whole, and particularly on the relation between the strain rate and stress deviators.

In all cases, however, the nondependence of relation (2.1) on $\dot{\varepsilon}$ and $\dot{\gamma}$ indicates that the viscosity effects noted in section 1 are attributable (given the usual assumption [9, 11] of nondependence of the volume strain on the quantity I_2) to the importance of the effect of strain rate on the volume compression of sands and clays. A similar conclusion was previously reached in connection with undisturbed clays as a result of direct measurements of the residual volume strains after an explosion and comparison with the strains at the shock front [5]. The values of the latter at the same distances from the explosion center proved to be much less than the residual strains. This cannot be explained within the framework of the elastic-plastic model [11], but is easily interpreted on the basis of the experimental results examined above.

The author thanks N. V. Zvolinskii and S. S. Grigoryan for their helpful advice and V. V. Mel'nikov for participating in the laboratory investigations.

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